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Iteration Learning Control for Uncertain Nonlinear Systems with Time-Varying Output Constraint ⋆

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Abstract: A novel Adaptive Iteration Learning Control (AILC) method is proposed to solve the trajectory tracking problem for a class of nonlinear uncertain systems with external disturbances. Furthermore, the output of system is required to be bounded by a time-varying function. To this end, a Barrier Lyapunov Function (BLF) term is integrated into the AILC scheme such that the impact of the uncertainties and disturbances are significantly reduced without violating the output constraints. A Barrier Composite Energy Function (BCEF) is utilized to analyze the convergence of state error and the boundedness of output. The validity of the proposed AILC scheme is verified by a numerical example. In addition, a high-fidelity simulation platform that can generate a real-life turbulent flow is utilized to demonstrate the robustness of the algorithm.

Keywords: Iterative and repetitive learning control, Control of constrained systems, Uncertain systems, Disturbance rejection, High fidelity simulation

1. INTRODUCTION

As a highly effective data-driven control methodology, Iteration Learning Control (ILC) has proven its success in addressing control problems with repetitive tasks. By obtaining input-output measurements with limited information about the previous iterations, ILC method can improve the tracking performance of the current iteration. Since first proposed by Arimoto et al. (1984), ILC has achieved significant development with successful applications in various engineering fields (Xu et al. (2008)), such as robot manipulators (Chien and Tayebi (2008)), spacecrafts (Wu et al. (2021)) and autonomous vehicles (Li et al. (2022)). Recently, ILC algorithms have also been applied to unmanned aerial vehicles (UAVs) (Meraglia and Lovera (2022); Foudeh et al. (2020); Cobb et al. (2019)). For example, for monitoring overhead power lines (Jones (2005)), ILC provides superior behavior in fault-finding operations and accurate trajectory tracking. It is worth noting that the aforementioned applications share not only the fast response requirement of their repetitive tasks but also some critical safety requirements which can be interpreted as time-varying output constraints. In addition, the processes to be controlled are usually nonlinear, inaccurately modeled and effected by time-varying external disturbances, for instance, the winds in the UAV case.

The adaptive ILC approaches are widely acknowledged for their abilities to handle system uncertainties by effectively utilizing information related to system structure and the repetitive operation patterns (Yu et al. (2016)). In recent years, some novel adaptive ILC methods (Xu and Tan (2002); Yu et al. (2016); Jin and Xu (2013); Yin et al. (2011); Yang et al. (2022)) have been developed by introducing adaptive mechanisms into iteration domain. However, only a few works (Jin and Xu (2013)) have considered the system with output constraints.

Many techniques have been developed in Non-ILC fields to handle the output constraint requirements, such as set invariance notions based method (Hu and Lin (2001); Liu and Michel (1994)), using artificial potential fields (Khatib (1986); Warren (1990)) and Barrier Lyapunov function (BLF) (Tee et al. (2009, 2011)), etc. Among them, BLF-based ones are widely recognized in practice. To integrate BLF into the ILC framework, Sebastian et al. (2019) considered a robotic manipulator system with hard output constraints. In Jin and Xu (2013), the authors discussed constant output constraints by utilizing the barrier composite energy function (BCEF). To the best of our knowledge, the time-varying output constraint problem for the system considered in this paper has not been addressed via ILC methods.

Inspired by the above discussions, a novel BLF-Based adaptive iteration learning control (AILC) scheme is proposed. The presented method achieves high-performance trajectory tracking for a certain class of uncertain nonlinear systems subject to stringent output constraints and perturbations generated by external disturbances. These uncertainties, constraints, and disturbances in consideration are all time-varying and could be state-dependent. In addition, the influence of initial errors is also taken into account. Distinguished from the aforementioned adaptive ILC, this new AILC not only can deal with parametric uncertainties and external disturbances, but also can handle

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time-varying output constraints. The reason to emphasize the merits of considering time-varying output constraints mainly lies in consideration of the possible large initial error. Note that, there are many techniques developed in ILC community to relax or remove the identical initial condition (i.i.c.), such as alignment condition (Xu and Jin (2013)) and time-varying boundary layer technique (Chien et al. (2004)). However, in the context of BLF, a large initial error either results in a conservative and large constraint setting (which is not very useful in practice) or draws too near to the boundary (which may lead to excessive control input) at the beginning of the process.

Notations: $\mathbb{R}^n$ denotes the sets of n dimensional real vectors; $\mathbb{N}$ denotes the field natural numbers; $|a|$ denotes the absolute value of the scalar a; $|b(t)|_{\sup}$ denotes the absolute maximum value of $b(t)$ in $[0,T]$; $\|x\|$ represents the Euclidean norm of the vector x; $\hat{(\cdot)}$ represents the estimate of $(\cdot)$; $\mathrm{RMS}((x(t)) = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}$. Other notations will be introduced as needed.

2. PRELIMINARY

Definition 1. (Tee et al. (2009)) A Barrier Lyapunov Function is a scalar function $V(x)$, defined with respect to the system $\dot{x} = f(x)$ on an open region $\mathcal{D}$ containing the origin, that is continuous, positive definite, has continuous first-order partial derivatives at every point of $\mathcal{D}$, has the property $V(x) \to \infty$ as x approaches the boundary of $\mathcal{D}$, and satisfies $V(x(t)) \leq b$, $\forall t \geq 0$ along the solution of $\dot{x} = f(x)$ for $x(0) \in \mathcal{D}$ and some positive constant b.

Lemma 1. (Tee et al. (2009)) For any positive constants k_1, k_2, let $\mathcal{Z}_1 := \{z_1 \in \mathbb{R} : -k_1 < z_1 < k_2\} \subset \mathbb{R}$ and $\mathcal{N} := \mathbb{R}^l \times \mathcal{Z}_1 \subset \mathbb{R}^{l+1}$ be open sets. Consider the system $\dot{\eta} = h(t,\eta)$, where $\eta := [w, z_1]^\top \in \mathcal{N}$, and $h : \mathbb{R}_+ \times \mathcal{N} \to \mathbb{R}^{l+1}$ is piecewise continuous in t and locally Lipschitz in z, uniformly in t, on $\mathbb{R}_+ \times \mathcal{N}$. Suppose that there exist functions $U : \mathbb{R}^l \to \mathbb{R}_+$ and $V_1 : \mathcal{Z} \to \mathbb{R}_+$, continuously differentiable and positive definite in their respective domains, such that

$$\begin{aligned} &V_1(z_1) \to \infty \quad z_1 \to -k_1 \quad \text{or} \quad z_1 \to k_2 \\ &\gamma_1(\|w\|) \geq U(w) \geq \gamma_2(\|w\|) \end{aligned}$$

where γ_1 and γ_2 are class K_∞ functions. Let $V(\eta) := V_1(z_1) + U(w)$, and $z_1(0)$ belong to the set $z_1 \in (-k_1, k_2)$. If the equation $\dot{V} = \frac{\partial V}{\partial \eta}\eta \leq 0$ holds: then $z_1(t)$ remains in the open set $z_1 \in (-k_1, k_2)$, $\forall t \in [0, \infty)$.

3. PROBLEM FORMULATION AND CONTROLLER DESIGN

Consider the following SISO model[1] with both parametric and unstructured uncertainties:

$$\begin{aligned} \dot{x}_{1,i}(t) &= x_{2,i}(t) \\ \dot{x}_{2,i}(t) &= \boldsymbol{\theta}(t)^\top \boldsymbol{\xi}(\mathbf{x}_i(t)) + b(u_i(t) + d(\mathbf{x}_i, t)) \\ y_i(t) &= x_{1,i}(t) \end{aligned} \tag{1}$$

where $i \in \mathbb{N}$ is the iteration index, $\mathbf{x}_i \triangleq [x_{1,i},\, x_{2,i}]^\top \in \mathbb{R}^2$ and $u_i(t) \in \mathbb{R}$ are accessible states and input, respectively. $\boldsymbol{\theta} \triangleq \boldsymbol{\theta}(t) \in \mathbb{R}^m$ is a time-varying unknown vector function. $\boldsymbol{\xi}_i \triangleq \boldsymbol{\xi}(\mathbf{x}_i(t)) \in \mathbb{R}^m$ denotes a known state-dependent vector function. $d_i \triangleq d(\mathbf{x}_i, t) \in \mathbb{R}$ represents an unknown unstructured uncertainty. $t \in [0,T]$ where $T > 0$ is the operation time in each iteration. $b \in \mathbb{R}$ is a known constant. The tracking error is defined as $\mathbf{e}_i \triangleq [e_{1,i},\, e_{2,i}]^\top = \mathbf{x}_r - \mathbf{x}_i \in \mathbb{R}^2$, where $\mathbf{x}_r$ stands for the desired state vector.

[1] For the sake of readability, we present our method for a simple second-order system. It is not difficult to extend the proposed algorithm to a high-order system which features a cascade integrator and can represent many nonlinear systems encountered in practice.

The system satisfies the following assumptions.

Assumption 1. There exist a reference output $y_r = x_{1,r}$ and a reference input u_r satisfying the system model:

$$\begin{aligned} \dot{x}_{1,r} &= x_{2,r} \\ \dot{x}_{2,i} &= \boldsymbol{\theta}^\top \boldsymbol{\xi}_r + b_r(u_r + d_r) \\ y_r &= x_{1,r} \end{aligned} \tag{2}$$

such that $|y_r(t)| < \varepsilon_r(t)$ holds for output constraint $\varepsilon_r(t)$.

Assumption 2. The alignment condition that requires the final states from the previous iteration are identical to the initial states of the current iteration, is satisfied in every iteration i.e., $\mathbf{x}_i(0) = \mathbf{x}_{i-1}(T)$, for all $i \in \mathbb{N}$. Furthermore, reference states are closed in space, meaning $x_r(0) = x_r(T)$. For tracking error, we have:

$$\mathbf{e}_i(0) = \mathbf{x}_r(0) - \mathbf{x}_i(0) = \mathbf{x}_r(T) - \mathbf{x}_{i-1}(T) = \mathbf{e}_{i-1}(T) \tag{3}$$

Assumption 3. The unstructured uncertainty $d(\mathbf{x}_i, t)$ is locally Lipschitz continuous, i.e., $|d_r - d_i| \leq \varphi(\mathbf{x}_r, \mathbf{x}_i)\|\mathbf{x}_r - \mathbf{x}_i\|$, in which $\varphi(\mathbf{x}_r, \mathbf{x}_i)$ and $d_r \triangleq d(\mathbf{x}_r, t)$ are known bounded functions.

Assumption 4. For unstructured uncertainty d_i and parametric uncertainty $\boldsymbol{\theta}_i = [\theta_{1,i},\, \theta_{2,i},\, \cdots,\, \theta_{m,i}]^\top$, there exist constants $\bar{d}_i$ and $\bar{\theta}_l$, $l = 1, 2, \cdots, m$, such that

$$|\theta_{l,i}|_{\sup} < \bar{\theta}_l \quad , \quad |d_i|_{\sup} < \bar{d}_i$$

Remark 1. As a projection operation will be needed in the proposed control algorithm, Assumption 4 is essential. However, one can always select sufficiently large parameters to bound these uncertainties without compromising the performance of the algorithm.

The control objective is to find a control input sequence $\{u_i\}_{i\in\mathbb{N}}$ such that the states $\mathbf{x}_i(t)$ converge to the desired $\mathbf{x}_r(t)$ without violating the constraints, by considering parametric uncertainties and unstructured uncertainties.

Remark 2. The problem addressed in this work is a constraint tracking problem. It is constrained in the sense that all the states of System (1) are observable, but only a part of them are equipped with size limitation. In fact, such scenario is rather prevalent in practical applications, for instance, for a manipulator working alongside human being, the safety of mankind is critical. The safety requirements then can be interpreted into joint angle constraints.

In order to achieve the control objective, the control law with ILC scheme in the i-th iteration is designed as

$$\begin{aligned} u_i = \frac{\partial V_{b,i}}{\partial e_{1,i}} + \gamma e_{2,i} + \frac{1}{b}[\dot{x}_{2,r} - \hat{\boldsymbol{\theta}}_i^\top \boldsymbol{\xi}_i] - \hat{d}_i \\ + \operatorname{sign}(p(\mathbf{e}_i)b)\varphi_i\|\mathbf{e}_i\| \end{aligned} \tag{4}$$

$$\hat{\boldsymbol{\theta}}_i = \mathcal{P}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{i-1}) - \rho p(\mathbf{e}_i)\boldsymbol{\xi}_i \qquad \hat{\boldsymbol{\theta}}_0 = 0 \tag{5}$$

$$\hat{d}_i = \mathcal{P}_d(\hat{d}_{i-1}) - \beta p(\mathbf{e}_i) b \qquad \hat{d}_0 = 0 \tag{6}$$

where $\frac{\partial V_{b,i}}{\partial e_{1,i}} = k\sec^2(\frac{\pi e_{1,i}^2}{2\varepsilon_b^2})e_{1,i}$ with $V_{b,i} = k\frac{2\varepsilon_b^2}{\pi}\tan(\frac{\pi e_{1,i}^2}{2\varepsilon_b^2})$. $V_{b,i}$ is a BLF that approaches infinity as $|e_{1,i}| \to \varepsilon_b(t)$, where $\varepsilon_b(t)$ denotes the constraint on output tracking error. k, γ are positive control gains, and $\rho, \beta > 0$ are adaptive learning gains, respectively. The function $p(\mathbf{e}_i)$ will be determined later, while $\mathcal{P}_{\boldsymbol{\theta}}$ and $\mathcal{P}_d$ represent the projection operations

$$\mathcal{P}_{\boldsymbol{\theta}}(\boldsymbol{\theta}_i) = [\mathcal{P}_{\boldsymbol{\theta}}(\hat{\theta}_{1,i}), \cdots, \mathcal{P}_{\boldsymbol{\theta}}(\hat{\theta}_{m,i})]^\top$$

$$\mathcal{P}_{\boldsymbol{\theta}}(\hat{\theta}_{l,i}) = \begin{cases} \hat{\theta}_{l,i}, & |\hat{\theta}_{l,i}| \le \bar{\theta}_l, \\ \mathrm{sign}(\hat{\theta}_{l,i})\bar{\theta}_l, & |\hat{\theta}_{l,i}| > \bar{\theta}_l, \end{cases} l = 1, \cdots, m$$

$$\mathcal{P}_d(\hat{d}_i) = \begin{cases} \hat{d}_i, & |\hat{d}_i| \le \bar{d}_i \\ \mathrm{sign}(\hat{d}_i)\,\bar{\mathrm{d}}_i, & |\hat{d}_i| > \bar{d}_i \end{cases}$$

Remark 3. In practice, the output constraints are typically specified with respect to the output signal y_i, rather than the error signal $e_{1,i}$. To develop an appropriate boundary function $\varepsilon_b(t)$, we assume that there is a safety-related requirement $k_b(t)$ imposed directly on $y_i(t)$, i.e., $|y_i(t)| < k_b(t)$. Then, according to the relations $|y_r(t)| < \varepsilon_r(t)$ and $|y_i(t)| < |y_r(t)| + |e_{1,i}(t)|$, we can select $\varepsilon_b(t)$ such that $0 < \varepsilon_b(t) < k_b(t) - \varepsilon_r(t)$. By ensuring $|e_{1,i}(t)| < \varepsilon_b(t)$, $|y_i(t)| < |y_r(t)| + |e_{1,i}(t)| < \varepsilon_b(t) + \varepsilon_r(t) < k_b(t)$ holds.

Remark 4. There exist several types of barrier functions available for designing a proper BLF-controller, among which the nature logarithmic functions $\log(\frac{\varepsilon_b^2}{\varepsilon_b^2 - \mathbf{e}^\top\mathbf{e}})$ and trigonometric tangent functions $\tan(\frac{\pi\mathbf{e}^\top\mathbf{e}}{2\varepsilon_b^2})$ are commonly used in the literature (Xu and Jin (2013)). In this work, we choose the tan-type barrier function, but the others, including the natural logarithmic type one, are also applicable, as long as they verify Lemma 2 that will be mentioned in the next section.

4. STABILITY ANALYSIS

To facilitate the analysis of state tracking error convergence under output constrained conditions, we introduce a simplified system:

$$\dot{x}_1 = x_2 \quad , \quad \dot{x}_2 = bv$$

where $b > 0$ is a positive coefficient for the control input, v is control input and $x_1, x_2 \in \mathbb{R}$ are system states. Let $\mathbf{x} \triangleq [x_1, x_2]^\top$, we define tracking error $\mathbf{e} = [e_1, e_2]^\top = \mathbf{x}_r - \mathbf{x}$, $|e_1(0)| < \varepsilon_b(0)$, where $\mathbf{x}_r$ is the vector of desired states. Consider the control input as follows:

$$v = k\sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2})e_1 + \gamma e_2 + \frac{1}{b}\dot{x}_{2,r} \tag{7}$$

where k, γ are positive gains.

First, we can obtain the error dynamics:

$$\dot{\mathbf{e}} = A_s\mathbf{e} + B\{-bk[\sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2}) - 1]e_1\}$$

$$A_s = \begin{bmatrix} 0 & 1 \\ -bk & -b\gamma \end{bmatrix} \quad , \quad B = [0\,1]^\top$$

Due to A_s is the Hurwitz matrix, there exist symmetric positive definite matrices $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$, $Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix}$, which satisfy Lyapunov equation $A_s^\top P + PA_s = -Q$. From Lyapunov equation, the elements of P, Q should satisfy the following relations

$$P_{11} = \frac{k + b\gamma^2}{2b\gamma k}Q_{11} + \frac{k}{2\gamma}Q_{22} - Q_{12} \tag{8}$$

$$P_{12} = \frac{1}{2kb}Q_{11} \quad , \quad P_{22} = \frac{1}{2b^2k\gamma}Q_{11} + \frac{1}{2b\gamma}Q_{22} \tag{9}$$

with positive elements Q_{11}, Q_{22}, P_{11}, P_{12}, P_{22}.

Consider the Lyapunov function candidate V_1

$$V_1 = V_0 + V_b \quad , \quad V_0 = \mathbf{e}^\top\bar{P}\mathbf{e}$$

$$V_b = \frac{2P_{22}bk\varepsilon_b^2}{\pi}\tan(\frac{\pi e_1^2}{2\varepsilon_b^2}) \tag{10}$$

where $\bar{P} = \begin{bmatrix} b\lambda P_{12} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$. From (8)-(9), it is trivial to verify $P_{22} > 0$, $b\lambda P_{12} > 0$, $b\lambda P_{12}P_{22} - P_{12}^2 = \frac{1}{4kb}Q_{11}Q_{22} > 0$, which indicates matrix $\bar{P}$ is symmetric positive definite.

The following analysis will demonstrate that the BLF is bounded and the asymptotic convergence of $\mathbf{e}$ is guaranteed. At first, taking the time derivative of (10) yields:

$$\begin{aligned} \dot{V}_1 =& \mathbf{e}^\top(A_s^\top P + PA_s)\mathbf{e} + 2(b\gamma P_{12} - P_{11})e_1e_2 \\ &+ 2\mathbf{e}^\top PB\{-bk[\sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2}) - 1]e_1\} + \dot{V}_b \\ =& -\mathbf{e}^\top Q\mathbf{e} + 2(bkP_{22} + b\gamma P_{12} - P_{11})e_1e_2 \\ &- 2bkP_{12}[\sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2})(1 + \frac{P_{22}\dot{\varepsilon}_b}{P_{12}\varepsilon_b}) - 1]e_1^2 \\ &+ \frac{4P_{22}bk\varepsilon_b}{\pi}\tan(\frac{\pi e_1^2}{2\varepsilon_b^2})\dot{\varepsilon}_b \end{aligned} \tag{11}$$

where $p(\mathbf{e}_i) = 2P_{12}e_{1,i} + 2P_{22}e_{2,i}$. In order to prove that $\dot{V}_1$ is negative definite, we proceed our analysis by considering the following two cases for $\dot{\varepsilon}_b$ separately.

Case 1: $\dot{\varepsilon}_b \le 0$**.** By substituting $\dot{\varepsilon}_b \le 0$, together with $\frac{4P_{22}bk\varepsilon_b}{\pi}\tan(\frac{\pi e_1^2}{2\varepsilon_b^2})\dot{\varepsilon}_b \le 0$, the equation (11) becomes

$$\begin{aligned} \dot{V}_1 \le& -\mathbf{e}^\top Q\mathbf{e} - 2bkP_{12}[\sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2})(1 + \frac{P_{22}\dot{\varepsilon}_b}{P_{12}\varepsilon_b}) - 1]e_1^2 \\ &+ 2(bkP_{22} + b\gamma P_{12} - P_{11})e_1e_2 \end{aligned}$$

In view of (8)-(9), we obtain

$$\dot{V}_1 \le -[Q_{11}\sec^2(\frac{\pi e_1^2}{\varepsilon_b^2})(1 + \frac{P_{22}\dot{\varepsilon}_b}{P_{12}\varepsilon_b})]e_1^2 - Q_{22}e_2^2$$

By choosing a suitable boundary function $\varepsilon_b(t)$ that satisfies $1 + \frac{P_{22}\dot{\varepsilon}_b}{P_{12}\varepsilon_b} > 0$, we have $\dot{V}_1 \le -\alpha_1 e^\top e$ with $\alpha_1 = \max\{[Q_{11}\sec^2(\frac{\pi e_1^2}{\varepsilon_b^2})(1 + \frac{P_{22}\dot{\varepsilon}_b}{P_{12}\varepsilon_b})], Q_{22}\}$.

Case 2: $\dot{\varepsilon}_b > 0$**.** Making use of (11), one obtains

$$\begin{aligned} \dot{V}_1 =& -\mathbf{e}^\top Q\mathbf{e} - 2bkP_{12}[\sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2}) - 1]e_1^2 \\ &+ 2(bkP_{22} + b\gamma P_{12} - P_{11})e_1e_2 \\ &+ \frac{4bkP_{22}\varepsilon_b\dot{\varepsilon}_b}{\pi}[\tan(\frac{\pi e_1^2}{2\varepsilon_b^2}) - \sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2})\frac{\pi e_1^2}{2\varepsilon_b^2}] \end{aligned}$$

Since $\frac{\pi e_1^2}{2\varepsilon_b^2} > 0$, we have $\tan(\frac{\pi e_1^2}{2\varepsilon_b^2}) - \sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2})\frac{\pi e_1^2}{2\varepsilon_b^2} < 0$. Meanwhile, by taking (8)-(9), it follows that

$$\dot{V}_1 \le -Q_{11}\sec^2(\frac{\pi e_1^2}{\varepsilon_b^2})e_1^2 - Q_{22}e_2^2 \le -\alpha_2 e^\top e$$

where $\alpha_2 = \max\{Q_{11}\sec^2(\frac{\pi e_1^2}{\varepsilon_b^2}), Q_{22}\}$.

Combining two cases, we obtain $\dot{V}_1 \le -\alpha e^\top e$ with $\alpha = \max\{\alpha_1, \alpha_2\}$. Now we summarize the above analysis in the following lemma.

Lemma 2. Consider the system in the form of

$$\dot{x}_1 = x_2 \quad , \quad \dot{x}_2 = bv$$

where $b > 0$ and v is the control input. Define $\mathbf{x} \triangleq [x_1, x_2]^\top \in \mathbb{R}^2$, $\mathbf{x}_r \triangleq [x_{r,1}, x_{r,2}]^\top \in \mathbb{R}^2$ and $\mathbf{e} \triangleq [e_1, e_2]^\top \triangleq \mathbf{x}_r - \mathbf{x}$ as system states, desired states and tracking error, respectively. Given the controller as

$$v = k\sec^2(\frac{\pi e_1^2}{2\varepsilon_b^2})e_1 + \gamma e_2 + \frac{1}{b}\dot{x}_{2,r} \ ,$$

there exists a BLF $V(\mathbf{e}) > 0$, $\forall \mathbf{e} \neq 0$, such that

$$\frac{\partial V}{\partial \mathbf{e}} g(\mathbf{e}) \le -\alpha \mathbf{e}^\top \mathbf{e}$$

with $\alpha, \gamma, k > 0\,, g(\mathbf{e}) = [e_2,\ -bk\sec(\frac{\pi e_1^2}{2\varepsilon_b^2})e_1 - b\gamma e_2]^\top$.

The main result is stated in Theorem 1 as follows.

Theorem 1. For System (1) that meets Assumptions 1-4, the control law (4) and the adaptive learning laws (5)-(6) guarantee that

i) $\mathbf{e}_i$ approaches to zero asymptotically along the iteration axis for $\forall t \in [0, T]$, namely $\lim_{i\to\infty} \|\mathbf{e}_i(t)\| = 0$.
ii) Further, if the initial state tracking error verifies the constraint, i.e., $|e_{1,i}(0)| < \varepsilon_b(0)$, then $|e_{1,i}(t)| < \varepsilon_b(t)$ will be guaranteed for all $t \in [0, T]$, $i = 1,\ 2,\ \cdots$.

Using Lemma 2 and controller (4), we have

$$\dot{\mathbf{e}}_i = g(\mathbf{e}_i) + f(\mathbf{e}_i) \tag{12}$$

$$g(\mathbf{e}_i) = [e_{2,i},\ -bk\sec(\frac{\pi e_{1,i}^2}{2\varepsilon_b^2})e_{1,i} - b\gamma e_{2,i}]^\top$$

$$f(\mathbf{e}_i) = [0,\ -(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i}})^\top \boldsymbol{\xi} - b(d_i - \hat{d}_i) - \mathrm{sign}(p(\mathbf{e}_i)b)b\varphi_i\|\mathbf{e}_i\|]^\top$$

Proof 1. The BCEF in i-th iteration is designed as

$$E_i(t) = V_{1,i} + V_{2,i} + V_{3,i} \tag{13}$$

$$V_{2,i} = \frac{1}{2\rho}\int_0^T (\theta - \hat{\theta}_i)^\top(\theta - \hat{\theta}_i)d\tau \tag{14}$$

$$V_{3,i} = \frac{1}{2\beta}\int_0^T (d_i - \hat{d}_i)^2 d\tau \tag{15}$$

where $V_{1,i} \triangleq V(\mathbf{e}_i(t))$ is a barrier Lyapunov function satisfying Lemma 1. Note that, although $V_{2,i}$ and $V_{3,i}$ contain the estimation errors of the uncertainties, it is unnecessary to show the convergence of the estimates, therefore no persistent excitation condition is required.

The proof is structured into three main parts. Part I establishes the non-increasing property of the proposed BCEF over the iteration domain at instant $t = T$. In Part II, we demonstrate the boundedness of BCEF at the first iteration and exhibit its finiteness throughout any iteration. Finally, in Part III, we obtain the convergence of the tracking error and the boundedness of output.

Part I: Difference of BCEF

Consider the difference of BCEF (13) between two adjacent iterations at time $t = T$

$$\begin{aligned}\Delta E_i(T) &= E_i(T) - E_{i-1}(T)\\ &= \Delta V_{1,i}(T) + \Delta V_{2,i}(T) + \Delta V_{3,i}(T)\end{aligned} \tag{16}$$

For the first term on (16), we have

$$\begin{aligned}\Delta V_{1,i}(T) &= V(\mathbf{e}_i(T)) - V(\mathbf{e}_{i-1}(T))\\ &= V(\mathbf{e}_i(0)) + \int_0^T (\frac{\partial V^\top}{\partial \mathbf{e}_i}\dot{\mathbf{e}}_i)d\tau - V(\mathbf{e}_{i-1}(T))\\ &= \int_0^T \frac{\partial V^\top}{\partial \mathbf{e}_i}\{g(\mathbf{e}_i) + f(\mathbf{e}_i)\}\\ &\le \int_0^T -\alpha \mathbf{e}_i^\top \mathbf{e}_i - p(\mathbf{e}_i)[(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i}})^\top \boldsymbol{\xi}_i + b(d_i - \hat{d}_i)\\ &\qquad + \mathrm{sign}(p(\mathbf{e}_i)b)b\varphi_i\|\mathbf{e}_i\|]\end{aligned} \tag{17}$$

when $p(\mathbf{e}_i) = 0$, the second and third terms of (17) become zero, then (17) becomes $\Delta V_{1,i}(T) \le \int_0^T -\alpha \mathbf{e}_i^\top \mathbf{e}_i d\tau$; when $p(\mathbf{e}_i) \neq 0$, we obtain

$$\begin{aligned}\Delta V_{1,i}(T) \le &-\int_0^T \alpha \mathbf{e}_i^\top \mathbf{e}_i d\tau - \int_0^T p(\mathbf{e}_i)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i}})^\top \boldsymbol{\xi} d\tau\\ &- \int_0^T p(\mathbf{e}_i)b(d_i - \hat{d}_i)d\tau - \int_0^T |p(\mathbf{e}_i)b|\varphi_i\|\mathbf{e}_i\|d\tau\end{aligned} \tag{18}$$

According to the adaptive updating law (5), and using the inequality $(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i-1}})^\top(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i-1}}) \ge (\boldsymbol{\theta} - \mathcal{P}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{\boldsymbol{i-1}}))^\top(\boldsymbol{\theta} - \mathcal{P}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{\boldsymbol{i-1}}))$, we can derive

$$\begin{aligned}&\frac{1}{2\rho}\{(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i}})^\top(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i}}) - (\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i-1}})^\top(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i-1}})\}\\ &\qquad \le p(\mathbf{e}_i)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i}})^\top \boldsymbol{\xi}_i - \frac{\rho}{2}p(\mathbf{e}_i)^2\|\boldsymbol{\xi}_i\|^2\end{aligned}$$

therefore, for $\Delta V_{2,i}(T)$, we have

$$\Delta V_{2,i}(T) \le \int_0^T p(\mathbf{e}_i)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_{\boldsymbol{i}})^\top \boldsymbol{\xi}_i d\tau \tag{19}$$

In terms of $(d_r - d_{i-1})^2 \ge (d_r - \mathcal{P}_d(\hat{d}_{i-1}))^2$, we have

$$\begin{aligned}\Delta V_{3,i}(T) &\le \frac{1}{2\beta}\int_0^T (d_r - \hat{d}_i)^2 - (d_r - \mathcal{P}_d(\hat{d}_{i-1}))^2 d\tau\\ &\le \int_0^T p(\mathbf{e}_i)b(d_r - \hat{d}_i)d\tau\end{aligned} \tag{20}$$

Hence, combining all terms from (18), (19) and (20) yields

$$\begin{aligned}\Delta E_i(T) \le &-\int_0^T \alpha \mathbf{e}_i^\top \mathbf{e}_i d\tau - \int_0^T |p(\mathbf{e}_i)b|\varphi_i\|\mathbf{e}_i\|d\tau\\ &+ \int_0^T p(\mathbf{e}_i)b(d_r - d_i)d\tau\end{aligned} \tag{21}$$

From Assumption 3 and $p(\mathbf{e}_i)b(d_r - d_i) \le |p(\mathbf{e}_i)b||d(\mathbf{x}_r, t) - d(\mathbf{x}_i, t)|$, one can $\Delta E_i(T) \le -\int_0^T \alpha \mathbf{e}_i^\top \mathbf{e}_i d\tau$, i.e., $E_i(T)$ is monotonically decreasing along iteration axis.

Part II: Boundedness and Finiteness of $E_i(t)$

To prove the boundedness of the given BCEF at the first iteration, we differentiate $E_1(t)$ with respect to time t

$$\dot{E}_1(t) = \dot{V}_{1,1} + \dot{V}_{2,1} + \dot{V}_{3,1}$$

Similar to (18), we can obtain

$$\begin{aligned}\dot{V}_{1,1} \le &-\alpha \mathbf{e}_1^\top \mathbf{e}_1 - p(\mathbf{e}_1)(\boldsymbol{\theta} + p(\mathbf{e}_1)\boldsymbol{\xi}_1)^\top \boldsymbol{\xi}_1\\ &- p(\mathbf{e}_1)b(d_1 + p(\mathbf{e}_1)b) - |p(\mathbf{e}_1)b|\varphi_1\|\mathbf{e}_1\|\end{aligned}$$

$$\dot{V}_{2,1} \le \frac{1}{2\rho}\|\boldsymbol{\theta}\|^2 + \frac{\rho}{2}p(\mathbf{e}_1)^2\|\boldsymbol{\xi}_1\|^2 + p(\mathbf{e}_1)\boldsymbol{\xi}_1\boldsymbol{\theta}$$

$$\dot{V}_{3,1} \le \frac{1}{2\beta}d_r^2 + \frac{\beta}{2}p(\mathbf{e}_1)^2 b^2 + p(\mathbf{e}_1)bd_r$$

Thus, we have

$$\dot{E}_i(t) \leq -\alpha \mathbf{e}_1^\top \mathbf{e}_1 + \frac{1}{2\rho}\|\boldsymbol{\theta}\|^2 + \frac{1}{2\beta}d_r^2 - \frac{\rho}{2}p(\mathbf{e}_1)^2\|\boldsymbol{\xi}_1\|^2 - \frac{\beta}{2}p(\mathbf{e}_1)^2 b^2 < \infty \quad (22)$$

and since $\boldsymbol{\theta}$ and d_r are finite with respect to finite t and $\mathbf{x}_r$, (22) indicates that $E_1(T)$ is finite.

For any iteration $i \geq 2$, we have

$$\dot{E}_i(t) = \dot{V}_{1,i} + \dot{V}_{2,i} + \dot{V}_{3,i}$$

Also, similar to (18), we have

$$\begin{aligned}
\dot{V}_{1,i} \leq& -\alpha \mathbf{e}_i^\top \mathbf{e}_i - p(\mathbf{e}_i)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_i)^\top \boldsymbol{\xi}_i - p(\mathbf{e}_i)b(d_i - \hat{d}_i) \\
&- |p(\mathbf{e}_i)b|\varphi_i\|\mathbf{e}_i\| \\
\dot{V}_{2,i} =& \frac{1}{2\rho}(\theta - \hat{\theta}_i)^\top(\theta - \hat{\theta}_i) \\
=& \frac{1}{2\rho}[\|\boldsymbol{\theta}\|^2 - 2\boldsymbol{\theta}^\top \mathcal{P}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{i-1}) + \|\mathcal{P}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{i-1})\|^2] \\
&+ p(\mathbf{e}_i)[\boldsymbol{\theta} - (\mathcal{P}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{i-1}) - p(\mathbf{e}_i)\boldsymbol{\xi}_i)]^\top \boldsymbol{\xi}_i - \frac{\rho}{2}p(\mathbf{e}_i)^2\|\boldsymbol{\xi}_i\|^2 \\
=& \frac{C_1}{2\rho} + p(\mathbf{e}_i)(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_i)^\top \boldsymbol{\xi}_i - \frac{\rho}{2}p(\mathbf{e}_i)^2\|\boldsymbol{\xi}_i\|^2 \\
\dot{V}_{3,i} =& \frac{1}{2\beta}(d_i - \hat{d}_i)^2 \\
=& \frac{1}{2\beta}[d_r^2 - 2d_r\mathcal{P}(\hat{d}_{i-1}) + \mathcal{P}(\hat{d}_{i-1})^2 + 2\beta p(\mathbf{e}_i)bd_r \\
&- 2\beta p(\mathbf{e}_i)b\mathcal{P}(\hat{d}_{i-1}) + \beta^2 p(\mathbf{e}_i)^2 b^2] \\
=& \frac{C_2}{2\beta} + p(\mathbf{e}_i)b(d_r - \hat{d}_i) - \frac{\beta}{2}p(\mathbf{e}_i)^2 b^2
\end{aligned}$$

with $C_1 \triangleq \|\boldsymbol{\theta}\|^2 - 2\boldsymbol{\theta}^\top \mathcal{P}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{i-1}) + \|\mathcal{P}_{\boldsymbol{\theta}}(\hat{\boldsymbol{\theta}}_{i-1})\|^2$ and $C_2 \triangleq d_r^2 - 2d_r\mathcal{P}(\hat{d}_{i-1}) + \mathcal{P}(\hat{d}_{i-1})^2$

Since $p(\mathbf{e}_i)b(d_r - \hat{d}_i) - p(\mathbf{e}_i)b(d_i - \hat{d}_i) - |p(\mathbf{e}_i)b|\varphi_i\|\mathbf{e}_i\| \leq 0$, we have

$$\dot{E}_i(t) = \dot{V}_{1,i} + \dot{V}_{2,i} + \dot{V}_{3,i} \leq \frac{C_1}{2\rho} + \frac{C_2}{2\beta} < \infty$$

The initial value of BCEF at i-th iteration $E_i(0) = V_{1,i}(\mathbf{e}(0))$. Since $\mathbf{e}_i(0) = \mathbf{e}_{i-1}(T)$, and $\mathbf{e}_{i-1}(T)$ can be proved to be bounded by showing BCEF is bounded in the $(i-1)$-th iteration. The boundedness of $\dot{E}_i(t)$ and $E_i(0)$ implies the boundedness of $E_i(t)$, hence the boundedness of $V_{1,i}$ will be guaranteed, therefore $|e_{1,i}(t)| < \varepsilon_b(t)$ will be ensured for any time in all iterations.

Part III: Convergence of State Tracking Error and Boundedness of System Output

Since $\Delta E_i(T) \leq 0$, BCEF at the i-th iteration is

$$\begin{aligned}
\lim_{k\to\infty} E_k(T) =& E_i(T) + \sum_{i=2}^{k} \Delta E_i(T) \\
\leq& E_1(T) - \lim_{k\to\infty} \sum_{i=2}^{k} \int_0^T \alpha \mathbf{e}_i^\top \mathbf{e}_i d\tau
\end{aligned}$$

In terms of the positiveness of $E_k(T)$ and finiteness of $E_1(T)$, $\lim_{k\to\infty} \sum_{i=2}^{k} \int_0^T \alpha \mathbf{e}_i^\top \mathbf{e}_i d\tau$ converges. Therefore, we have $\lim_{k\to\infty} \sum_{i=2}^{k} \int_0^T \alpha \mathbf{e}_i^\top \mathbf{e}_i = 0$, which implies $\lim_{k\to\infty} \|\mathbf{e}_i(t)\| = 0$, $\forall t \in [0, T]$.

Hence, it follows that the output tracking error $e_{1,i}(t)$ in any iteration cannot exceed the open set $\varepsilon_b(t)$. Since $|y_i| < |y_r| + |e_{1,i}|$ and $0 < \varepsilon_b(t) < k_b(t) - \varepsilon_r(t)$, one concludes that $|y_i| < |y_r| + |e_{1,i}| < \varepsilon_b(t) + \varepsilon_r(t) < k_b(t)$. As a result, the boundedness of $e_{1,i}(t)$ and $y_r(t)$ implies that $y_i(t)$ is also bounded by the predefined constraint.

5. SIMULATION RESULTS

5.1 Numerical Example

Consider System (1) with $\mathbf{x}_1(0) = [-\frac{\pi}{3}, 0]^\top$ and $b = 0.897$. Reference output to be tracked is $y_r = \sin(\frac{\pi}{2}t)$, for $t \in [0, 12]$. $\boldsymbol{\theta}(t)$ is chosen to be time-varying, as $\bar{\theta}_l(t) = 1.2\text{sign}(\sin(\frac{2\pi}{3}t))$, $l = 1, 2, 3$ and $\boldsymbol{\xi}_i = [x_{1,i}, x_{1,i}^2, x_{1,i}^3]^\top$. The external disturbance is $d_i = 0.2\sin(x_{1,i}) + 0.2\sin(20\pi t)$. The tunning parameters are selected as $k = 2$, $\gamma = 4$, $\rho = 0.1$, $\beta = 0.1$, $p(\mathbf{e}_i) = 2.5e_{1,i} + 2e_{2,i}$, $\bar{d} = 2$ and $\bar{\theta}_l = 4$, for $l = 1, 2, 3$.

The sampling time of the simulation is $0.01s$. Under consistent setting, we consider two types of the output constraint: 1) the constant $\varepsilon_b = 1.2$; 2) $\varepsilon_b(t) = 1.1e^{-1.5t} + 0.1$. The simulation results are shown in Figs. 1-4.

The variation of $|e_{1,i}|_{\text{sup}}$, along the iteration axis, is shown in Fig. 1, which captures the worst performance of output in each iteration. This means that if $|e_{1,i}|_{\text{sup}}$ converges, the output trajectory also converges. Fig. 2 shows the convergence profile of the state tracking error. The convergence is achieved in both cases without violating the output constraints. Fig. 1 and Fig. 2 suggest that the time-varying error bound $\varepsilon_b(t)$ can enhance the convergence speed.

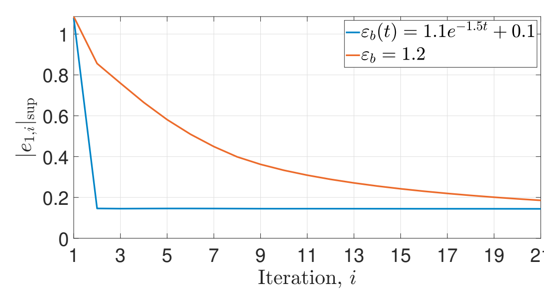

Fig. 1. Absolute maximum value of output error for $\varepsilon_b(t) = 1.1e^{-1.5t} + 0.1$ and $\varepsilon_b = 1.2$ in each iteration

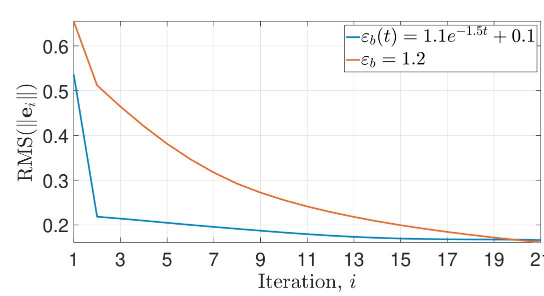

Fig. 2. RMS of the state error for $\varepsilon_b(t) = 1.1e^{-1.5t} + 0.1$ and $\varepsilon_b = 1.2$ in each iteration

By comparing Fig. 3 with Fig. 4, we observe that the output trajectory y_i in Fig. 4, under the application of time-varying error bound, tracks the reference trajectory better during the first iteration. Conversely, the output trajectory y_1 in Fig. 3 goes through a significant overshoot, which may pose safety issues in practical applications.

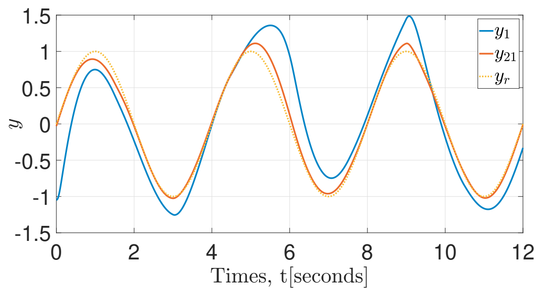

Fig. 3. Output Trajectory y_i for $\varepsilon_b = 1.2$

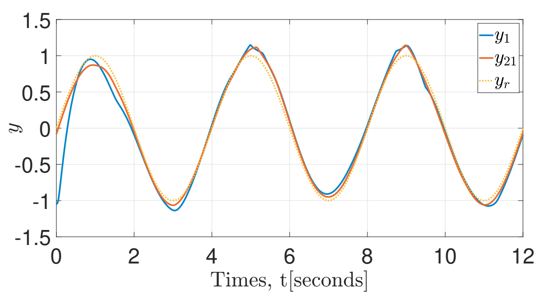

Fig. 4. Output Trajectory y_i for $\varepsilon_b(t) = 1.1e^{-1.5t} + 0.1$

5.2 High Fidelity Simulation

In this section, high fidelity simulation platform is used to verify the reliability and validity of our method based on a recently proposed GPU-optimized lattice Boltzmann solver (Li et al. (2020)). The platform can simulate two-way coupling in an efficient and scalable manner, even for turbulent flows, by a novel low-dissipation and low-dispersion fluid solver.

The experiment is implemented on a computer with an Nvidia RTX 3080Ti GPU. The rendering rate, grid resolution, and air viscosity are set as 100 fps, 500×250 and $0.0001\,N \cdot s/m^2$, respectively. In addition, we consider a uniform flow from top to bottom with speed $0.3\,m/s$.

To emphasize the influence of the complex fluid environment on the plant, a square object is chosen with length as $0.2\,m$ and weight as $1\,kg$. In this platform, the proposed method is applied to a 2D attitude control task of the square object. The bottom of this object has an angle-controllable nozzle that is able to generate a fluid with velocity ranging from $0\,m/s$ to $4\,m/s$. Moreover, to show the robustness of the controller, we choose the reference trajectory and initial states as follows

$$\mathbf{x}_r = [0.4\cos(\frac{\pi}{2}t),\, -\frac{\pi}{5}\sin(\frac{\pi}{2}t)]^\top \quad , \quad \mathbf{x}_1(0) = [0.4,\, 0]^\top$$

By using this trajectory, nozzle wil generate a high-velocity fluid at the begining of the simulation, which will cause great effect on the environment and further influence the object itself. The tunning parameters are the same as the above numerical example except for $T = 4$ and $\varepsilon_b(t) = 1.1e^{-0.4t} + 0.2$.

The RMS of the state error is shown in Fig. 6. It can be seen that, under the influence of turbulent flow, the controller is still able to keep the output of the object within the constraint and achieve convergence of the state error with iterations.

Remark 5. This object can be easily replaced with other UAV models for more realistic aircraft simulation tests. The external disturbances include steady winds, turbulent flows, various types of wind shear, and the propeller vortex. These influences are challenging to be quantified. Nevertheless, under comparable environmental conditions, wind disturbances on repetitive control assignments, such as the aforementioned UAV task, can be regarded as both time-varying and state-dependent.

6. CONCLUSION

This study proposes a novel AILC scheme for nonlinear systems with output constraints and uncertainties, which actively confines the output to a proper time-varying function throughout all iterations. Furthermore, this method guarantees the asymptotic convergence of the states to their desired values under alignment conditions, despite the presence of parametric and unstructured uncertainties which both are time-varying and state-dependent. Numerical examples and high-fidelity simulation results confirm the effectiveness and robustness of the proposed AILC scheme. Future studies will focus on exploring the extension of the considered systems to MIMO systems and applying our method to practical systems, especially UAV systems.

REFERENCES

Arimoto, S., Kawamura, S., and Miyazaki, F. (1984). Bettering operation of Robots by learning. *J. robot. syst.*, 1(2), 123–140.

Chien, C.J., Hsu, C.T., and Yao, C.Y. (2004). Fuzzy system-based adaptive iterative learning control for nonlinear plants with initial state errors. *IEEE Trans Fuzzy Syst.*, 12(5), 724–732.

Chien, C.J. and Tayebi, A. (2008). Further results on adaptive iterative learning control of robot manipulators. *Autom.*, 44(3), 830–837.

Cobb, M., Barton, K., Fathy, H., and Vermillion, C. (2019). An iterative learning approach for online flight path optimization for tethered energy systems undergoing cyclic spooling motion. In *IEEE ACC*, 2164–2170.

Foudeh, H.A., Luk, P., and Whidborne, J. (2020). Application of Norm Optimal Iterative Learning Control to Quadrotor Unmanned Aerial Vehicle for Monitoring Overhead Power System. *Energies*, 13(12), 3223.

Hu, T. and Lin, Z. (2001). *Control Systems with Actuator Saturation: Analysis and Design.* Springer Science & Business Media.

Jin, X. and Xu, J.X. (2013). Iterative learning control for output-constrained systems with both parametric and nonparametric uncertainties. *Autom.*, 49(8), 2508–2516.

Jones, D. (2005). Power line inspection - a UAV concept. In *The IEE Forum on Auton. Syst.*, 8–pp.

Khatib, O. (1986). Real-time obstacle avoidance for manipulators and mobile robots. In *Auton. robot vehicles*, 396–404. Springer.

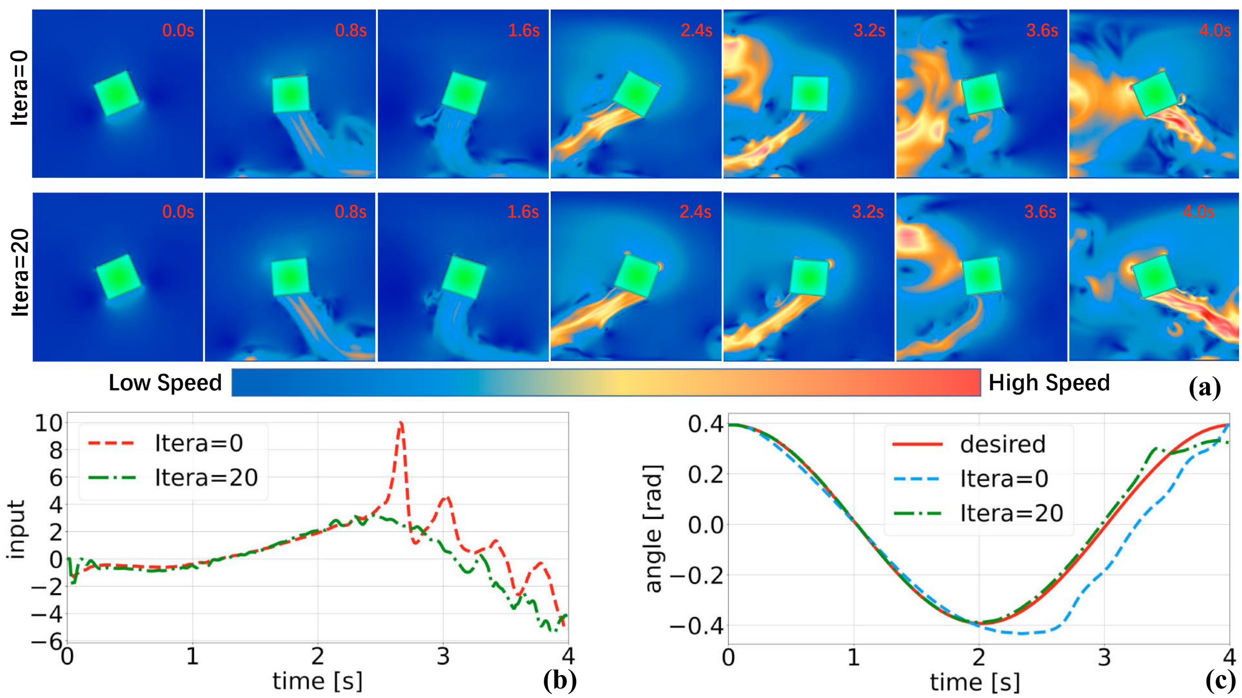

Fig. 5. **CFD Results.** The high-fidelity simulation results of the proposed method on the 2D-attitude control. (a) The snapshots of the high fidelity simulation in 20-th iteration. (b) The control inputs of the proposed algorithm. (c) The time history of the attitude angle $x_{1,1}$ of the flying agent.

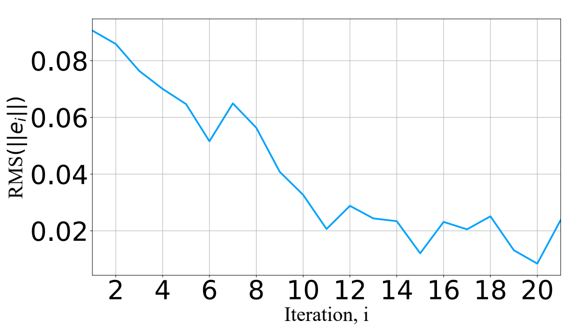

Fig. 6. RMS of the state error for $\varepsilon_b(t) = 1.1e^{-0.4t} + 0.2$ in each iteration

Li, W., Chen, Y., Desbrun, M., Zheng, C., and Liu, X. (2020). Fast and scalable turbulent flow simulation with two-way coupling. *ACM Trans. Graph.*, 39(4).

Li, X., Liu, C., Chen, B., and Jiang, J. (2022). Robust Adaptive Learning-Based Path Tracking Control of Autonomous Vehicles Under Uncertain Driving Environments. *IEEE trans Intell Transp Syst*, 1–12.

Liu, D. and Michel, A.N. (1994). *Dynamical systems with saturation nonlinearities: analysis and design*. Springer.

Meraglia, S. and Lovera, M. (2022). Smoother-Based Iterative Learning Control for UAV Trajectory Tracking. *IEEE Contr. Syst. Lett.*, 6, 1501–1506.

Sebastian, G., Li, Z., Tan, Y., and Oetomo, D. (2019). Analysis and experimental verification of a current-cycle iterative learning control for robotic manipulators with output constraints. In *IEEE CDC*, 7468–7473.

Tee, K.P., Ge, S.S., and Tay, E.H. (2009). Barrier Lyapunov Functions for the control of output-constrained nonlinear systems. *Autom.*, 45(4), 918–927.

Tee, K.P., Ren, B., and Ge, S.S. (2011). Control of nonlinear systems with time-varying output constraints. *Autom.*, 47(11), 2511–2516.

Warren, C. (1990). Multiple robot path coordination using artificial potential fields. In *IEEE Int. Conf. Robot. Autom.*, 500–505 vol.1.

Wu, Y.Y., Zhang, Y., and Wu, A.G. (2021). Preassigned finite-time attitude control for spacecraft based on time-varying barrier Lyapunov functions. *Aerosp Sci Technol*, 108, 106331.

Xu, J.X. and Jin, X. (2013). State-constrained iterative learning control for a class of MIMO systems. *IEEE Trans. Automat. Contr.*, 58(5), 1322–1327.

Xu, J.X., Panda, S.K., and Lee, T.H. (2008). *Real-Time Iterative Learning Control: Design and Applications*. Springer Science & Business Media.

Xu, J.X. and Tan, Y. (2002). A composite energy function-based learning control approach for nonlinear systems with time-varying parametric uncertainties. *IEEE Trans. Automat. Contr.*, 47(11), 1940–1945.

Yang, Z., Wang, W., Zhang, Y., Yan, Q., and Cai, J. (2022). High-Order Internal Model Based Barrier Iterative Learning Control for Time-Iteration-Varying Parametric Uncertain Systems With Arbitrary Initial Errors. *IEEE Access*, 10, 17619–17628.

Yin, C., Xu, J.X., and Hou, Z. (2011). An ilc scheme for a class of nonlinear continuous-time systems with time-iteration-varying parameters subject to second-order internal model. *Asian J Control*, 13(1), 126–135.

Yu, Q., Hou, Z., and Chi, R. (2016). Adaptive iterative learning control for nonlinear uncertain systems with both state and input constraints. *J Franklin Inst*, 353(15), 3920–3943.